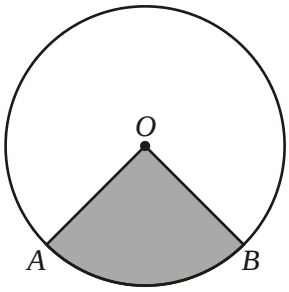
1. *O* is the centre of the circle which has a radius of 5.4 cm.



The area of the shaded sector *OAB* is 21.6 cm^2. Find the length of the minor arc *AB*.

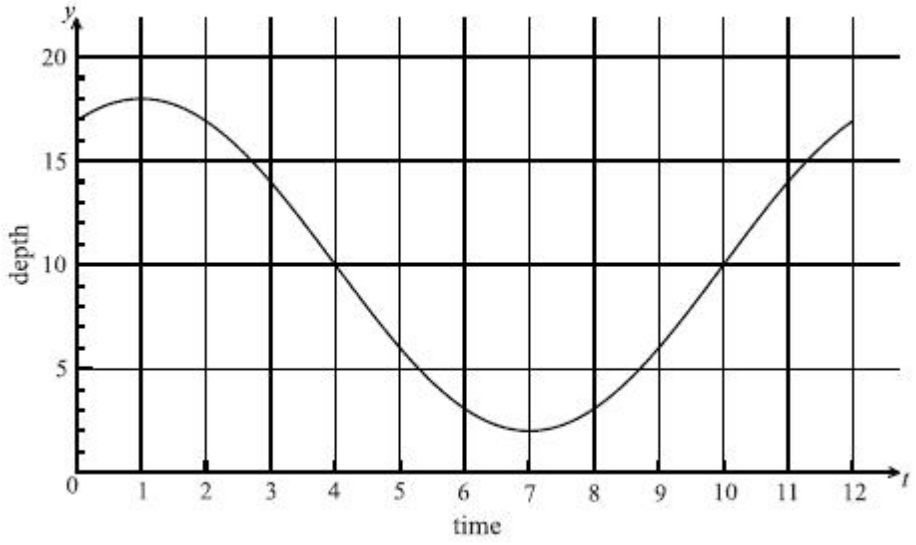
Working:

Answer:

.....

(Total 4 marks)

2. The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

(3)

(b) The depth of water can be modelled by the function $y = A \cos (B (t – 1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B.

(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P.

(2)

(Total 11 marks)

3. Solve the equation $3\sin^2 x = \cos^2 x$, for $0° \leq x \leq 180°$.

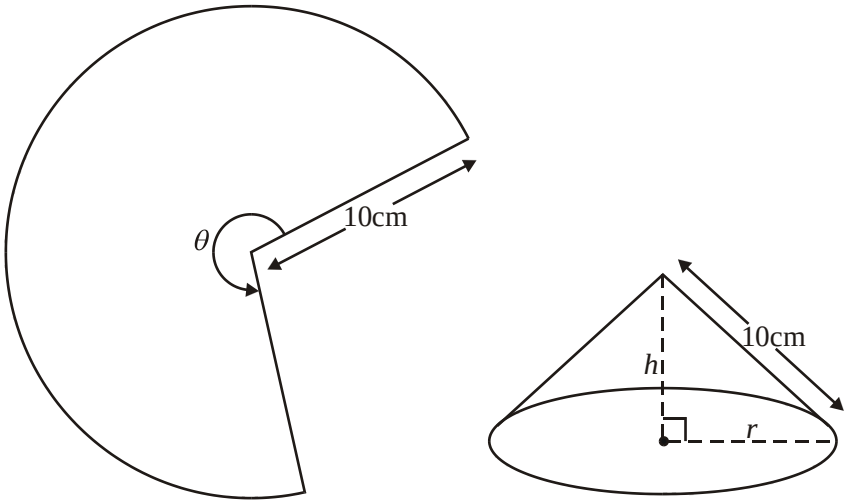
Working:

Answer:

.....

(Total 4 marks)

4. The diagrams show a circular sector of radius 10 cm and angle θ radians which is formed into a cone of slant height 10 cm. The vertical height h of the cone is equal to the radius r of its base. Find the angle θ radians.



Working:

Answer:

.....

(Total 4 marks)

5. The depth, y metres, of sea water in a bay t hours after midnight may be represented by the function

$$y = a + b\cos\left(\frac{2\pi}{k}t\right),$$ where a, b and k are constants.

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of

(a) a;

(b) b;

(c) k.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 4 marks)

6. Let $f(x) = \sin 2x$ and $g(x) = \sin(0.5x)$.

(a) Write down

(i) the minimum value of the function f;

(ii) the period of the function g.

(b) Consider the equation $f(x) = g(x)$.

Find the number of solutions to this equation, for $0 \le x \le \frac{3\pi}{2}$.

Working:

Answers:

(a) (i) .....

(ii) .....

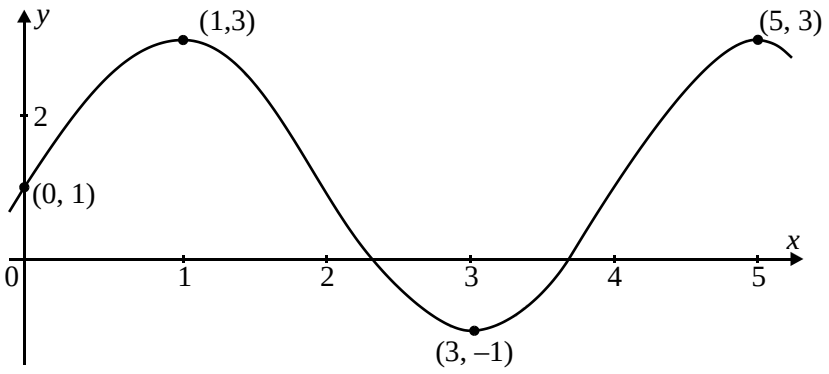
(b) .....

(Total 6 marks)

7. The diagram shows the graph of the function f given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \le x \le 5$, where A and B are constants, and x is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a) Write down the values of $f(1)$ and $f(5)$.

(2)

(b) Show that the period of f is 4.

(2)

The point (3, –1) is a minimum point of the graph.

(c) Show that $A = 2$, and find the value of B.

(5)

(d) Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(4)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \leq x \leq 5$.

(e) Find

(i) the point where this tangent meets the curve;

(ii) the value of k.

(6)

(f) Solve the equation $f(x) = 2$ for $0 \leq x \leq 5$.

(5)

(Total 24 marks)

8. Consider the following statements

A: $\log_{10}(10^x) > 0$.

B: $-0.5 \leq \cos(0.5x) \leq 0.5$.

C: $-\frac{\pi}{2} \leq \arctan x \leq \frac{\pi}{2}$.

(a) Determine which statements are true for all real numbers x. Write your answers (yes or no) in the table below.

Statement	(a) Is the statement true for all real numbers x? (Yes/No)	(b) If not true, example
A		
B		
C		

(b) If a statement is not true for all x, complete the last column by giving an example of one value of x for which the statement is false.

Working:

(Total 6 marks)

9. Given that $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$ and $0° \leq \theta \leq 360°$,

(a) find the value of θ;

(b) write down the **exact** value of $\tan \theta$.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

10. $f(x) = 4 \sin\left(3x + \frac{\pi}{2}\right)$.

For what values of k will the equation $f(x) = k$ have no solutions?

Working:

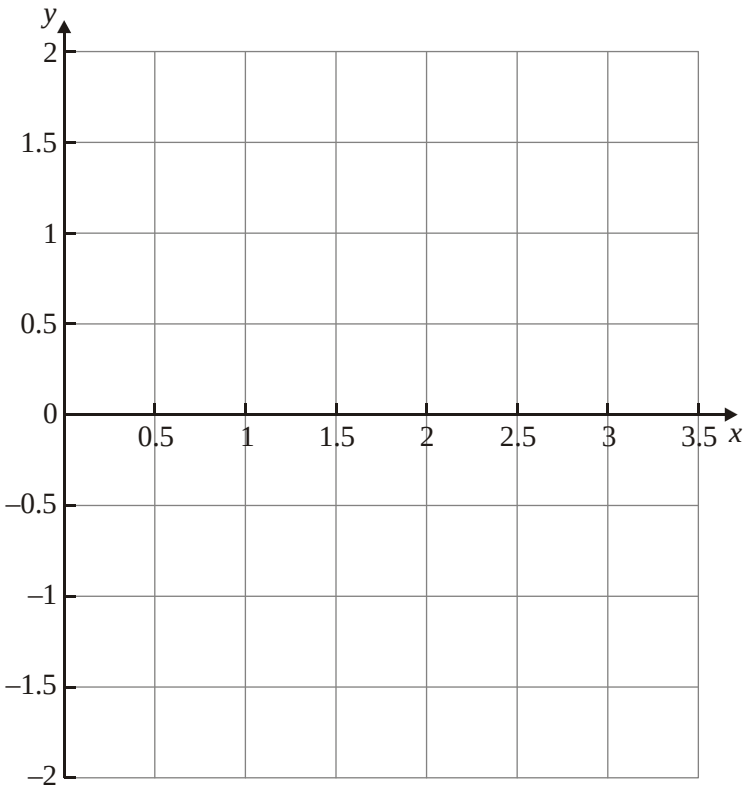
Answer:

.....

(Total 4 marks)

11. Let $f(x) = \sin(2x + 1)$, $0 \leq x \leq \pi$.

(a) Sketch the curve of $y = f(x)$ on the grid below.



(b) Find the x-coordinates of the maximum and minimum points of $f(x)$, giving your answers correct to one decimal place.

Working:

Answer:

(b) ……..

(Total 6 marks)

12. Let $f(x) = \frac{1}{2}\sin 2x + \cos x$ for $0 \leq x \leq 2\pi$.

(a) (i) Find $f'(x)$.

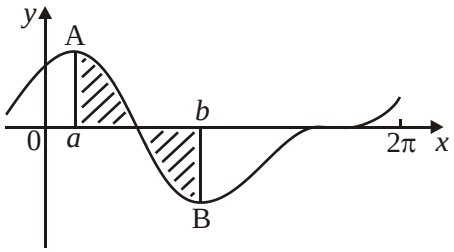
One way of writing $f'(x)$ is $-2\sin^2 x - \sin x + 1$.

(ii) Factorize $2\sin^2 x + \sin x - 1$.

(iii) Hence or otherwise, solve $f'(x) = 0$.

(6)

The graph of $y = f(x)$ is shown below.



There is a maximum point at A and a minimum point at B.

(b) Write down the x-coordinate of point A.

(1)

(c) The region bounded by the graph, the x-axis and the lines $x = a$ and $x = b$ is shaded in the diagram above.

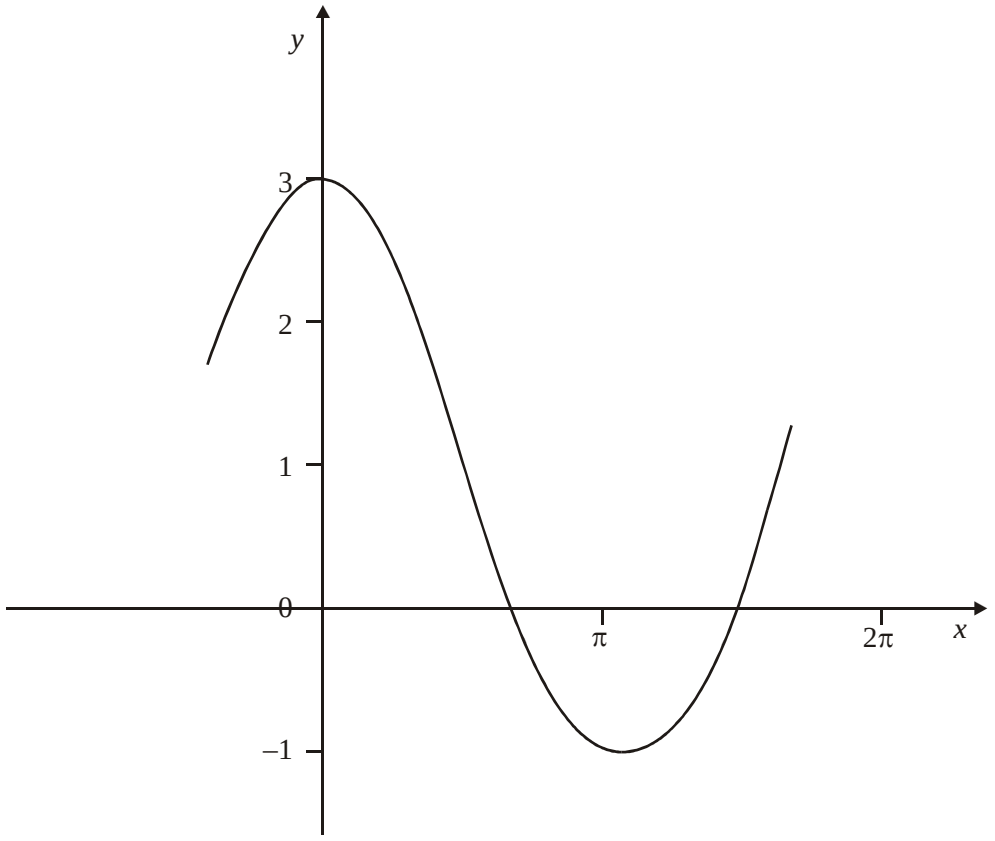
(i) Write down an expression that represents the area of this shaded region.

(ii) Calculate the area of this shaded region.

(5)

(Total 12 marks)

13. Part of the graph of $y = p + q \cos x$ is shown below. The graph passes through the points (0, 3) and $(\pi, -1)$.



Find the value of

(a) p;

(b) q.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

14. Consider $g(x) = 3 \sin 2x$.

(a) Write down the period of g.

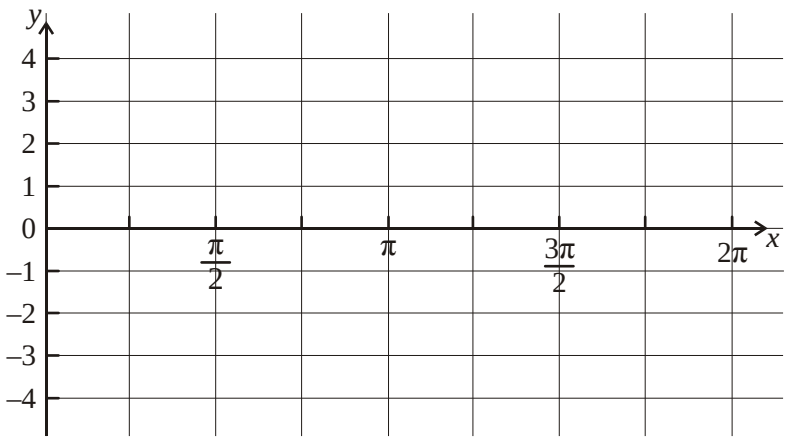
.....

.....

.....

(1)

(b) On the diagram below, sketch the curve of g, for $0 \leq x \leq 2\pi$.



(3)

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \leq x \leq 2\pi$.

.....

.....

.....

.....

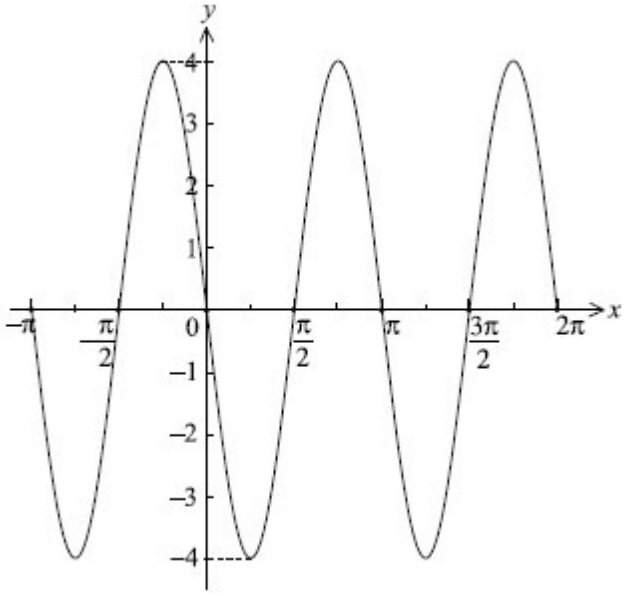
.....

.....

(2)

(Total 6 marks)

15. Let $f(x) = a \sin b(x - c)$. Part of the graph of f is given below.



Given that a, b and c are positive, find the value of a, of b and of c.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(Total 6 marks)

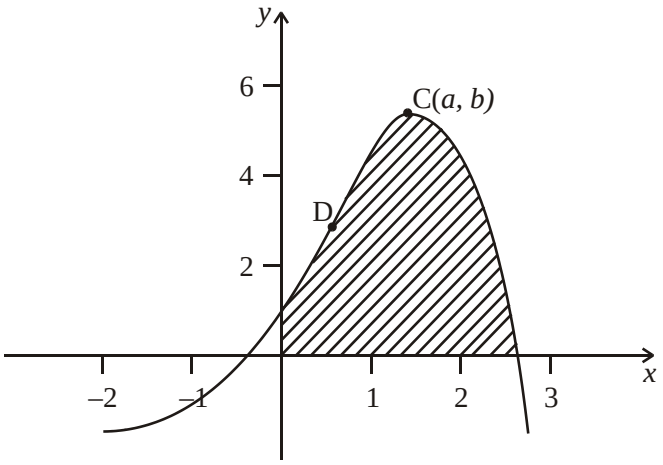
16. Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f(-\frac{\pi}{4}) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

(3)

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \leq x \leq 3$. The graph has a maximum turning point at C(a, b) and a point of inflexion at D.



(b) Find $\frac{dy}{dx}$.

(3)

(c) Find the **exact** value of a and of b.

(4)

(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(5)

(e) Find the area of the shaded region.

(2)
(Total 17 marks)

17. ***Note: Radians are used throughout this question.***

A mass is suspended from the ceiling on a spring. It is pulled down to point P and then released. It oscillates up and down.

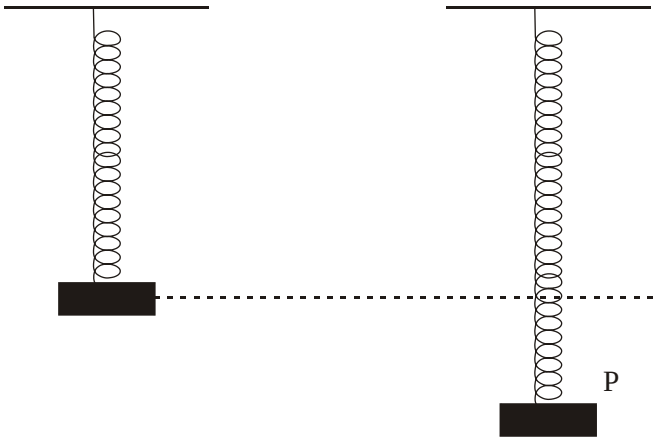


diagram not to scale

Its distance, s cm, from the ceiling, is modelled by the function $s = 48 + 10 \cos 2\pi t$ where t is the time in seconds from release.

(a) (i) What is the distance of the point P from the ceiling?

(ii) How long is it until the mass is next at P?

(5)

(b) (i) Find $\frac{ds}{dt}$.

(ii) Where is the mass when the velocity is zero?

(7)

A second mass is suspended on another spring. Its distance r cm from the ceiling is modelled by the function $r = 60 + 15 \cos 4\pi t$. The two masses are released at the same instant.

(c) Find the value of t when they are first at the same distance below the ceiling.

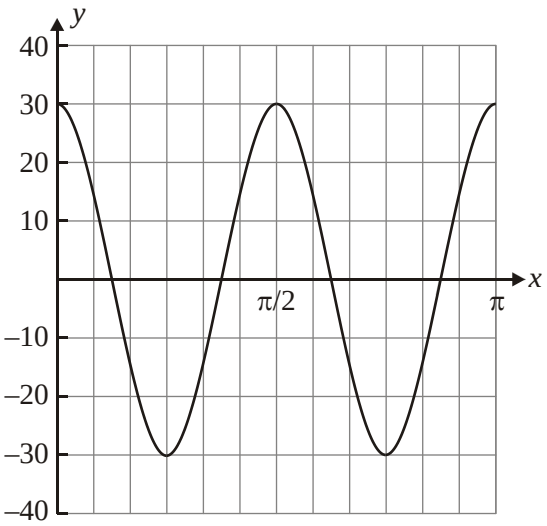
(2)

(d) In the first three seconds, how many times are the two masses at the same height?

(2)

(Total 16 marks)

18. The graph of a function of the form $y = p \cos qx$ is given in the diagram below.



(a) Write down the value of p.

(b) Calculate the value of q.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

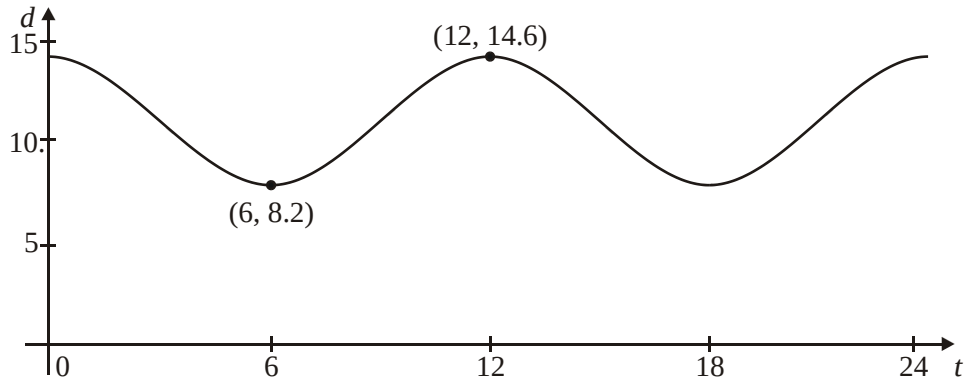
.....

(Total 6 marks)

19. A formula for the depth d metres of water in a harbour at a time t hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where P and Q are positive constants. In the following graph the point (6, 8.2) is a minimum point and the point (12, 14.6) is a maximum point.



(a) Find the value of

(i) Q;

(ii) P.

(3)

(b) Find the **first** time in the 24-hour period when the depth of the water is 10 metres.

(3)

(c) (i) Use the symmetry of the graph to find the **next** time when the depth of the water is 10 metres.

(ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep.

(4)